Transport of magnetic turbulence in SNR

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Abstract

Supernova remnants are known as sources of galactic cosmic rays for their non-thermal emission of radio waves, X-rays, and gamma-rays. However, the observed cosmic ray spectra are hard to reproduce within the standard acceleration theories based on the assumption of Bohm diffusion and steady-state calculations. We point out that a time-dependent treatment of the acceleration process together with a self-consistent treatment of the scattering turbulence is necessary. Therefore we numerically solve the coupled system of transport equations for cosmic rays and isotropic Alfveǹic turbulence. The equations are coupled through the growth rate of the turbulence determined by the cosmic-ray gradient and the spatial diffusion coefficient of cosmic rays given by the spectral energy density of the turbulence. The system is solved on a co-moving expanding grid extending upstream for dozens of shock radii, allowing for self-consistent study of cosmic-ray diffusion in the vicinity of their acceleration site. The transport equation for cosmic rays is solved in a test-particle approach based on pre-calculated hydro models. We demonstrate that the system is typically not in a steady state. In fact, even after several thousand years of evolution, no equilibrium situation is reached. The resulting time-dependent particle spectra strongly differ from those derived assuming a steady state and Bohm diffusion. The turbulence spectra show that Bohm-like diffusion is achieved only in a small energy band. Our results indicate that proper account for the evolution of scattering turbulence is crucial for the formation of the observed soft spectra.

1. Particle acceleration

We solve the time-dependent transport equation in spherically-symmetric 1-D geometry:

$$\frac{\partial N}{\partial t} = \nabla(D_r \nabla N - \vec{u}N) - \frac{\partial}{\partial p}\left((N\dot{p}) - \frac{\nabla \vec{u}}{3}Np\right) + Q \qquad (1)$$

- N Differential Number density of cosmic rays
- p Particle momentum
- $\vec{u}$ Advection velocity
- $\dot{p}$ Energy losses
- Q Source of thermal particles

Properties:

- Injection: thermal leakage model as proposed by [1]
- Solved in a co-moving, shock-centered frame with $(x-1) = (x^*-1)^3$ and $x = r/R_{sh}$
- very good resolution close to the shock, grid extending to several tens of shock radii into the upstream region ⟶ keeps all injected particles in simulation domain, self-consistent escape
- Sedov-Taylor stage: analytic expressions for flow parameters and magnetic field[2, 3]

One crucial parameter is the diffusion coefficient [4]. The diffusion coefficient

$$D_r = \frac{4v}{3\pi} r_g \frac{U_m}{\mathcal{E}_w} \qquad (2)$$

is usually assumed to be Bohm-like, i.e., $\frac{U_m}{\mathcal{E}_w} = \frac{\pi}{4}$, but scattering turbulence must be established first [5]. Here $\mathcal{E}_w$ denotes the energy density per unit logarithmic bandwidth of Alfven waves resonant with particles of momentum p according to $k = \frac{qB_0}{pc}$, U_m the energy density of the background magnetic field B_0 and r_g the gyro-radius of the cosmic rays with momentum p. The Bohm-diffusion approach is then equivalent to a featureless flat magnetic-turbulence spectrum [6, 7].

2. Magnetic turbulence

The transport of isotropic Alfvenic turbulence can be described by the following advection-diffusion equation for the spectral energy density of the waves, $E_w(r,k,t)$:

$$\frac{\partial E_w}{\partial t} + u \cdot (\nabla E_w) + C_W(\nabla \cdot u)E_w + k\frac{\partial}{\partial k}k^2 D_k \frac{\partial}{\partial k}\frac{E_w}{k^3} = 2(\Gamma_g - \Gamma_d)E_w \,. \qquad (3)$$

- C_W Prefactor for wave compression at the shock
- D_k Diffusion coefficient in wavenumber space
- Γ_g/Γ_d Growth and damping rates

This equation accounts for:

- Wave propagation along the background field
- Wave compression at the shock with $C_W = 1.5$
- Wave cascading with $D_k = k^5 v_A \sqrt{\frac{E_w}{2B_0^2}}$, resulting in a Kolmogorov like spectrum
- Wave growth by resonant amplification [6, 8]: $\Gamma_g = \frac{v_A p^2 v}{3E_w}\left|\frac{\partial N}{\partial r}\right|$
- Damping due to neutral-charged collisions [6, 9] and Ion-Cyclotron damping[10]

For computational reasons we use enhanced ISM-like turbulence as a seed spectrum.

3. Turbulence spectra

The turbulence spectra exhibit a complicated shape that varies with distance from the shock.

Spectra at the SNR shock:

- Region of extended growth ⟶ particles of all energies present, low energy particles dominate
- Break where cascading starts dominating, classical Kolmogorov power-law turbulence at higher k
- Wide, plateau-like region ⟶ similar to Bohm-diffusion

Spectra 15% upstream:

- Particle distribution peaked around escape energy ⟶ drives growth at resonant k
- Peak k corresponds to cutoff k at r shock
- No low-energy particles that far ahead of the shock
- High k-regime purely cascading dominated

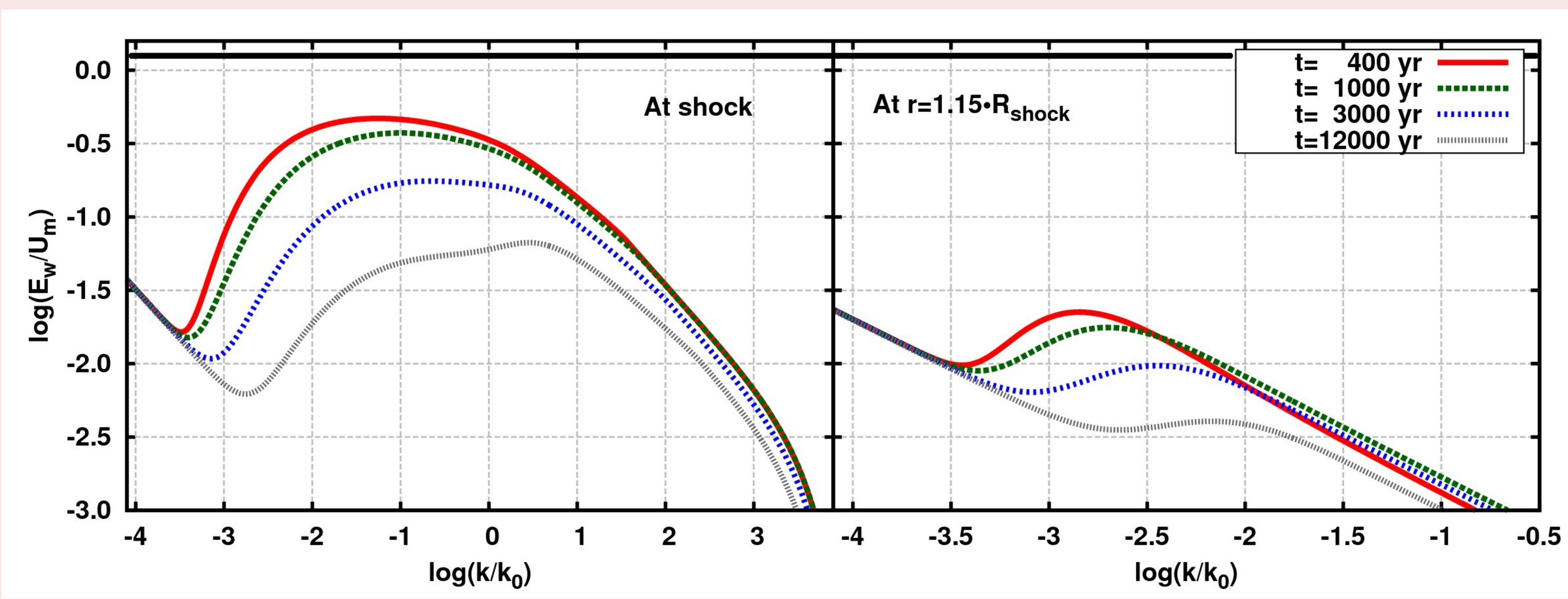

Figure: Left: Turbulence spectra, E_w, at the forward shock of a SNR at the age of 400 (red), 1000 (green), 3000 (blue) and 12000 (grey) years. Right: E_w at the distance $r = 1.15 \cdot R_{shock}$ for the same times. Particles with kinetic energy of 1 GeV are resonant with waves of wavenumber k_0. The black line corresponds to a Bohm-like turbulence spectrum.

Note, both plots show that **there is no steady state reached during our simulation.**

4. Particle spectra

We compare particle spectra obtained in the self-consistent coupled treatment of magnetic turbulence and cosmic rays to our previous results based on assumption of Bohm diffusion[11].

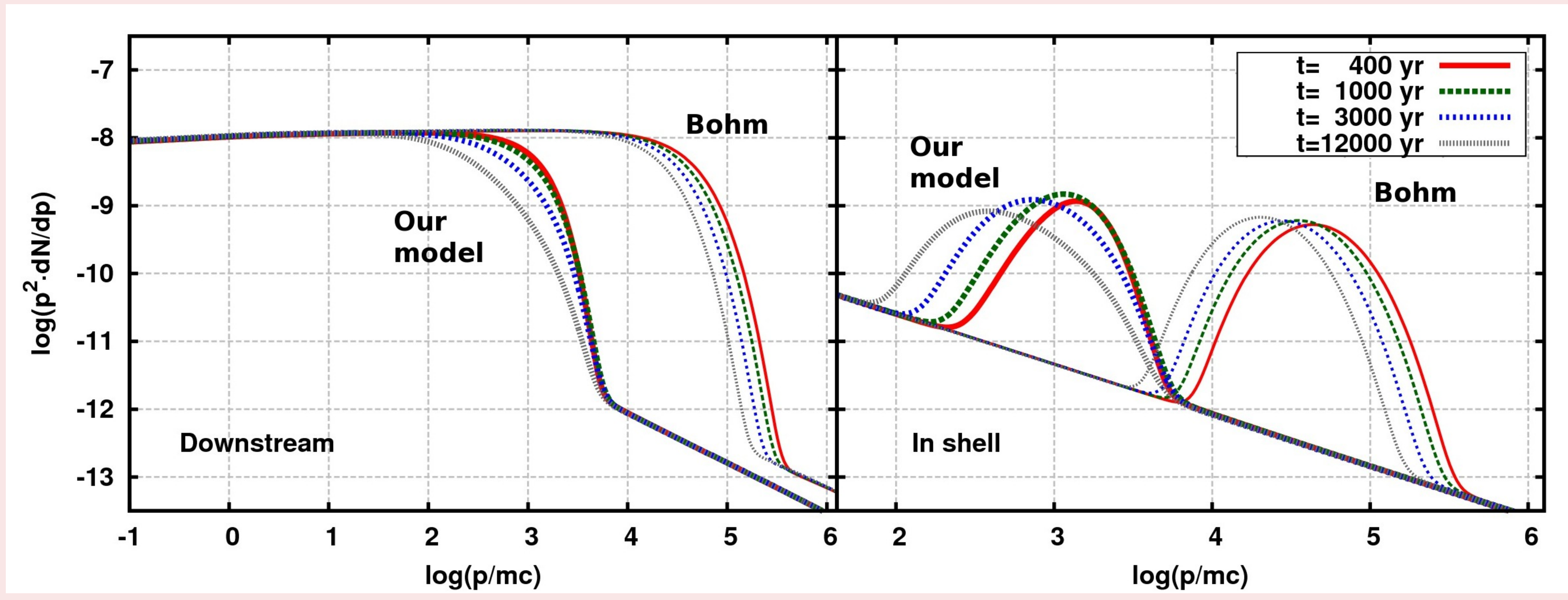

Figure: Left: the volume-integrated downstream cosmic-rays spectra for a SNR with an age of 400 (red), 3000 (blue) and 12000 (grey) years. Right: volume-integrated spectra in a shell of 0.5 pc thickness at $r = 1.15 \cdot R_{shock}$ for the same times. Both the self-consistent treatment (thick lines) and Bohm-like diffusion (thin lines) are presented.

Spectra at the SNR shock:

- Power-law spectra with lower cutoff-energy for self-consistent treatment
- Softer cutoff, sub exponential, spectral softening in cutoff region with time
- Faster drop in maximum energy for self-consistent case

Spectra 15% upstream:

- In the beginning: log-parabola for both cases [12]
- Constant shape in Bohm-case, permanent evolution in self-consistent case
- General: Broadening and shift to lower energies ⟶ result of shift in cutoff-energy at the shock

In general, the self-consistent treatment introduces a connection between the maximum energy and the injection parameter. To reach higher energies more particles have to be injected. **Thus there is a non-linear connection between the injection parameter, the cutoff energy and the normalisation of the cosmic-ray spectrum. Observational data might constrain the injection parameter.**

5. Conclusions

We developed a model for particle acceleration in SNR by solving the time-dependent transport equations for magnetic turbulence and cosmic rays. Our approach is 1-D and limited to the test-particle regime. We consider the cosmic rays being scattered by isotropic, Alfvenic turbulence that is subject to compression, advection, cascading, damping and growth due to resonant amplification of Alfven waves. We found that even for old remnants there is no steady state reached. Even after more than 10000 years both turbulence and particle spectra are still evolving. The need to continously develop magnetic turbulence upstream of the shock introduces nonlinearity in addition to that imposed by cosmic-ray feedback. Enhanced escape in the self-consistent treatment gives rise to the formation of softer spectra at late stages of the SNR evolution similar to those observed in high-energy gamma-rays. The maximum energy of cosmic rays tends to be lower than is estimated in earlier steady-state models.

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