A (semi)-analytical view of the inner structure of Pulsar Wind Nebulae

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BACKGROUND

NUMERICAL MODELS:

• Pulsar Wind Nebulae (PWNe) are successfully modelled by numerical simulations, in either 2 or 3-D (e.g. Del Zanna et al. 2004, Olmi et al. 2014; Porth et al. 2014).

• Simulations are however heavy in terms of computing time, which makes problematic tuning the models to best fit actual cases of PWNe.

• All these models are time-dependent. Observations show that real PWNe are time dependent as well, but it would be interesting to know whether a steady state solution could exist, and what would be the shape of the pulsar wind Termination Shock (TS) at equilibrium.

ANALYTICAL MODELS:

• The best known attempt to model analytically the structure of a PWN (by Kennel & Coroniti, 1980), although containing all the basic physical ingredients, fails because it assumes spherical symmetry; while in order to obtain a realistic modelling, at least 2-D models are required.

• Begelman & Li (1992), in the limit of negligible inertial effects, derived a 2-D analytical model that succeeds in reproducing, to some extent, the spatial distribution of the pressure inside the PWN.

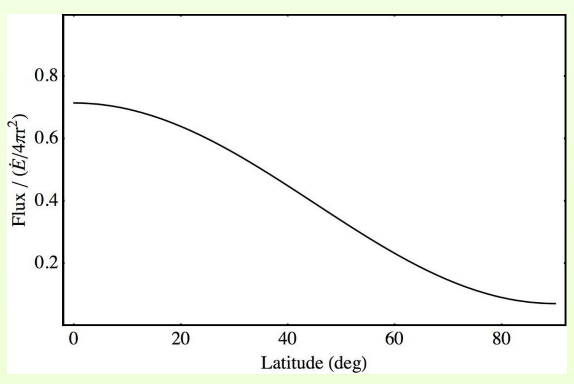
MAIN GOAL

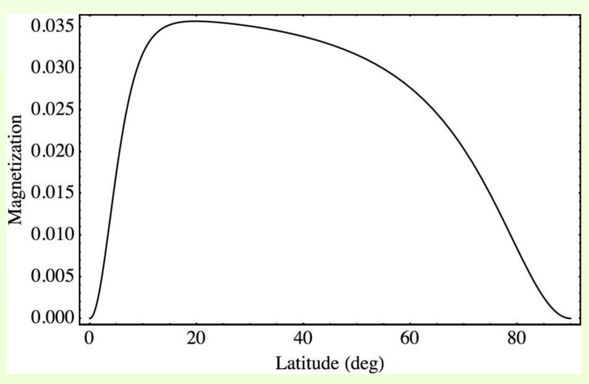
To Develop, in 2-D, a Kennel & Coroniti-like semi-analytic modelling

THE METHOD

Once assumed:

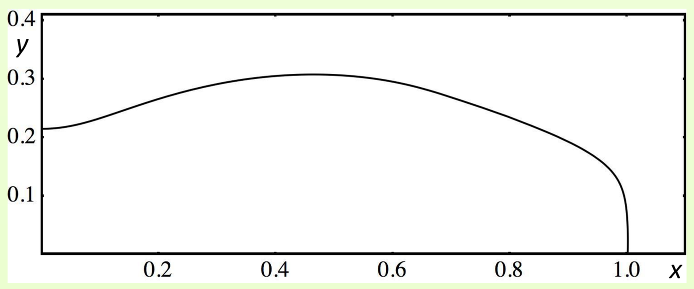
• a law for the anisotropic, magnetized, and relativistic pulsar wind





AND

• a shape for the TS (time average, from a numerical simulation)



We proceed as follows:

• by evaluating the immediate downstream MHD quantities, using the jump equations for relativistic magnetized oblique shocks (see e.g. Komissarov & Lyutikov 2011);

$\rho u_{\|} = \mathrm{const}; \qquad b\, u_{\|} = \mathrm{const}$

$(w + b^2/4\pi)\gamma u_{\|} = \mathrm{const};$

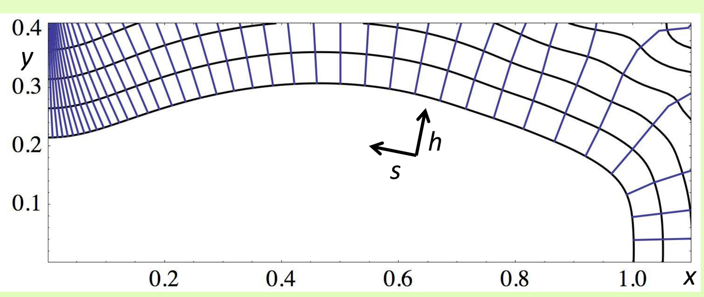
$(w + b^2/4\pi)u_{\|}^2 + p + b^2/8\pi = \mathrm{const}$

$(w + b^2/4\pi)u_{\|}u_{\perp} = \mathrm{const};$

with $w =$ enthalpy, $\{u_{\|}, u_{\perp}\}$; 4-velocity components

ρ and b density and magn. field in the comoving frame

• by defining a suitable set of orthogonal curvilinear coordinates (*s*: parallel to the TS surface; *h*: perpendicular to the TS surface), which considerably simplifies the form of the relativistic MHD equations;

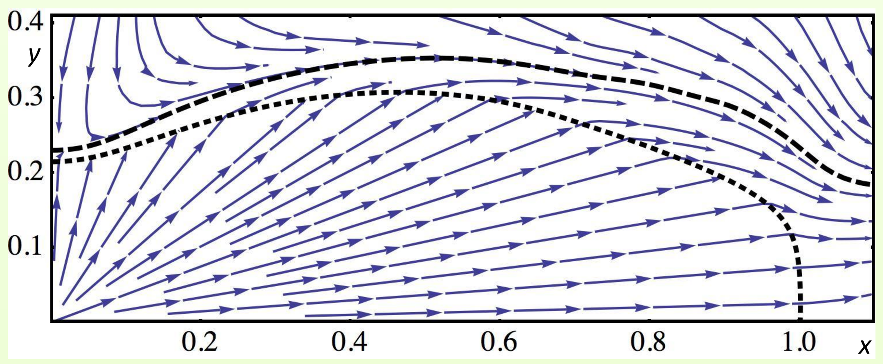


• by solving the system of relativistic MHD equations to the first order in the *h* coordinate; the differential equations then transform into a set of linear algebraic equations, which can be readily solved to get the first derivatives.

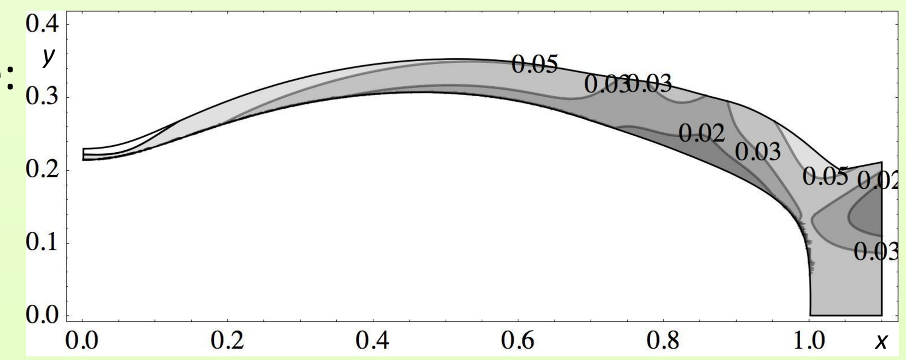
Some details on the method: the small-ρ limit is adopted; the primary variables p, u_h, u_s ($=u_{\|}, u_{\perp}$ on the shock) are all linearly expanded in *h*, while γ is computed exactly from them; as for the magnetic field, it resulted more convenient to first compute the linear solution for the magnetic flux components $\{bu_h, bu_s\}$, and then derive *b* from them.

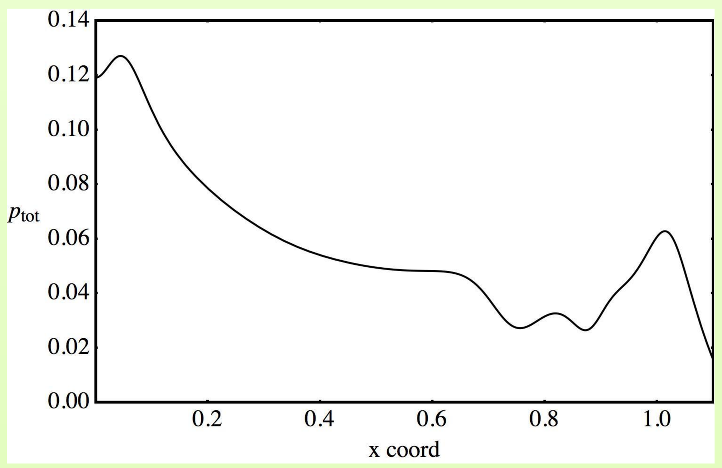
SOME RESULTS ON THE MHD SOLUTION

• The flow pattern:
- the downstream flow matches very well the average flow, from numerical models;
- the outer flow is simply an artifact of the linear expansion (the outer "unphysical" domain will not be considered below);
- nonetheless, the boundary surface (dashed line) closely matches the separation between inner and outer PWN, as in the numerical models.



• Contour plot of the total pressure:





• The profile of the total pressure at the boundary surface must match the pressure in the outer PWN.

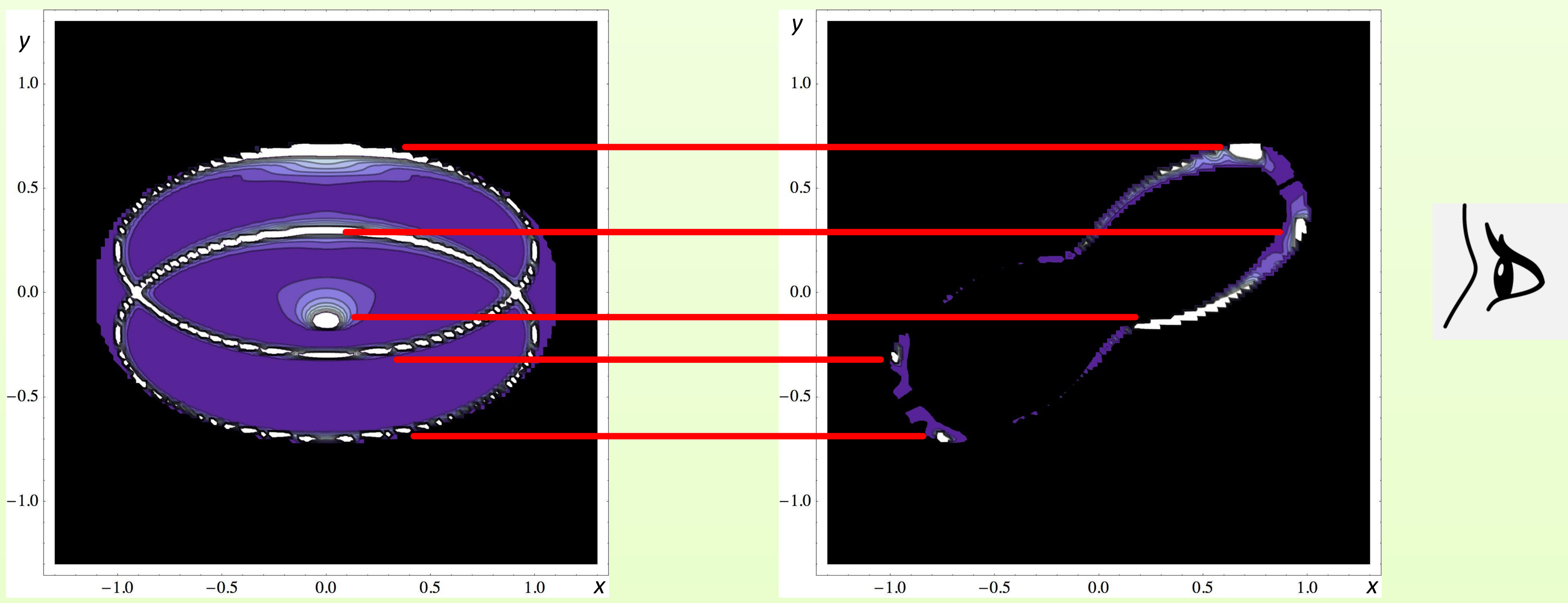
THE OUTER BOUNDARY CONDITIONS: Apparently, this solution does not depend on outer boundary conditions, but just on the shape of the TS. In fact that shape is determined, by numerical models, by solving the MHD equations with given conditions at the outer PWN boundary. While by now this semi-analytic approach needs an input from numerical models, in consideration of the short computing time required, a straightforward implementation would be to self-determine the shape of the TS, if the outer pressure at the boundary surface is known (possibly by an analytic model similar to that in Begelman & Li 1992).

SIMULATED MAPS

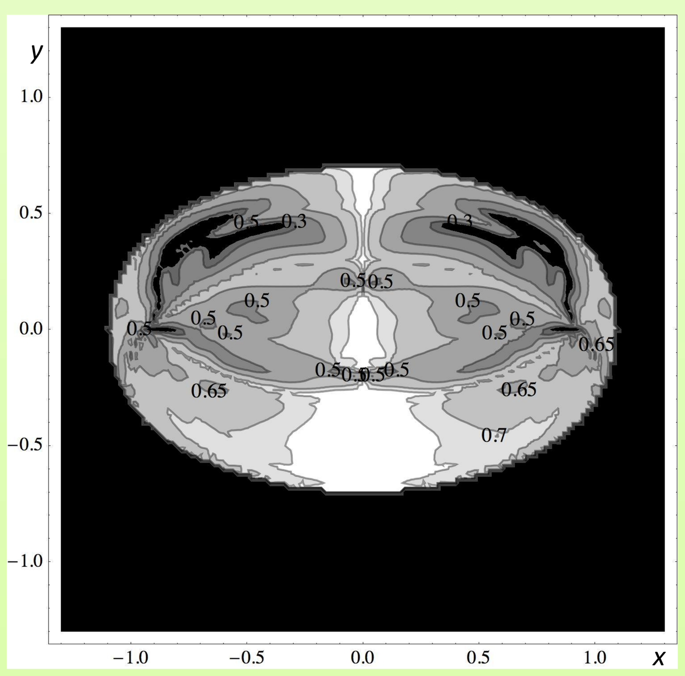
(aspect angle = 60°; power-law index = 2.2)

Total intensity map: (arbitrary units; linear scale)

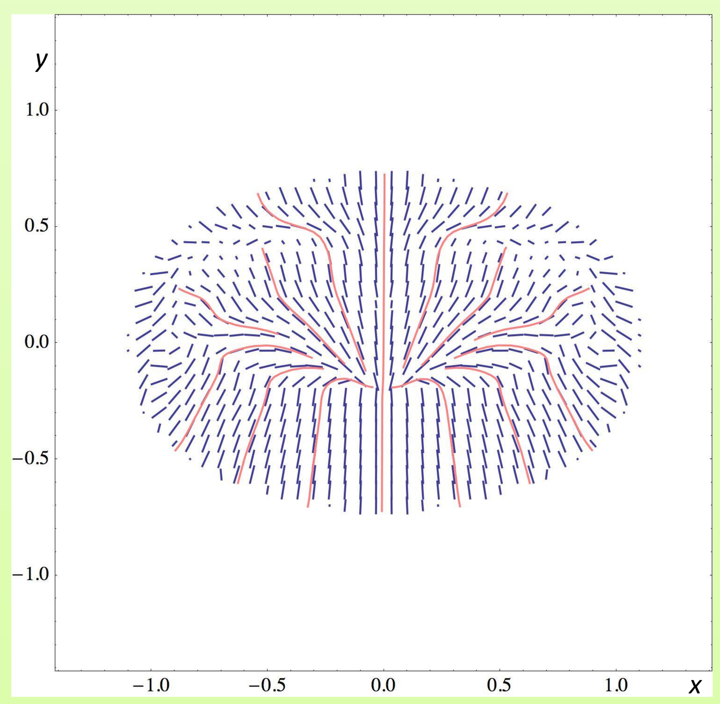
A slice of emissivities on the symmetry axis: (Doppler boosted according to observer's orientation)



Polarization fraction:



Vector map of the (electric) polarization: (red streamlines are added to allow a more immediate visualization of the pattern)



FINAL REMARKS:

• This semi-analytical approach is not intended to substitute numerical models, but rather to complement them.

• Even though it is known that time variability and deviations from the axial symmetry are real features, this modelling allows one to get a better insight on how a steady-state solution (which cannot be obtained by the usual numerical simulations) would be like.

• This approach is very light in terms of computing time, and therefore it may be useful as a tool for preliminary analyses in the parameter space, to best fit the inner structure of some PWNe.

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