Kinematic evolution of non-radiative supernova remnants

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Introduction

In this poster, we present new analytical formula describing the kinematic evolution of a non-radiative supernova remnants (SNRs). We constrain our discussion to the simplest situation, spherical expansion in homogeneous medium (no clouds) with negligible ambient pressure to obtain analytical solution. Thermal conduction, magnetic field and acceleration of cosmic ray particles are also not taken into account for simplification.

The kinematic evolution of a non-radiative SNR is characterized by the transition from a free expansion (FE) solution in the early time when the ejecta mass M_{ej} >> M_{SW} to the self similar Sedov-Taylor (ST) solution (Taylor 1946; Sedov 1959) in the late time when M_{SW} >> M_{ej}. Based on dimensional analysis, simple approximation by connecting the FE solution and ST solution is derived analytically. The analytical approximation is compared with numerical simulation and model in Truelove&McKee 1999 for different density profile in both ejecta and ambient medium, and found to be consistent with simulation results within a few percent accuracy for all the cases investigated in the work.

Analytical method

Based on π theorem (see e.g. chapter 1 of Barenblatt 1996), a physical relation involving k+m physical variables with k independent physical dimensions could be simplified into an physical relation with only m independent dimensionless quantities, i.e.

$$f(a_1,\ldots,a_k,\ldots,a_{k+m})=0 \quad (1)$$

involving k independent physical dimensions is equivalent to

$$F(\Pi_1,\ldots,\Pi_m)=0 \quad (2)$$

where Π_1, ..., Π_m are independent dimensionless quantities built with combination of $a_1,\ldots,a_{k+m}$.

The kinematic evolution of a non-radiative SNR under our simplified assumptions involves 5 different physical variables, explosion energy E_{SN}, ejecta mass M_{ej}, ambient medium density ρ_a, remnant age t and blast wave radius R_b, i.e. our goal is to derive an analytical approximation for the physical relation

$$f(E_{SN},M_{ej},\rho_a,t,R_b)=0. \quad (3)$$

The problem has 3 independent physical dimensions, length, time and mass. According to π theorem, above equation is equivalent to the following relation

$$F(\Pi_1, \Pi_2)=0 \quad (4)$$

where $\Pi_1=R_b^2M_{ej}/t^2E_{SN}$ and $\Pi_2=R_b^5\rho_a/E_{SN}t^2$ are the 2 independent dimensionless quantities available for the problem. Interestingly, $\Pi_1=\lambda^2$ and $\Pi_2=\xi$, where λ and ξ are appropriate dimensionless constants, actually provides the FE solution and ST solution respectively. Since the remnant follows the FE solution in the early time, i.e. $F(\Pi_1, \Pi_2)=\Pi_1-\lambda^2=0$ when t->0, and the ST solution in the late time, i.e. $F(\Pi_1, \Pi_2)=\Pi_2-\xi=0$ when t->∞. The following form of approximation

$$F(\Pi_1, \Pi_2)=(\Pi_1/\lambda^2)^\alpha+(\Pi_2/\xi)^\alpha-1=0 \quad (5)$$

is investigated in detail and compared with simulation, where α is a free parameter derived in fitting.

We assume the ejecta density profile has a flat core and a power law envelope, i.e.

$$\rho_{ej}(r<r_{core})=\rho_{core} \text{ and } \rho_{ej}(r\geq r_{core})=\rho_{core}(r/r_{core})^{-n}. \quad (6)$$

The ambient medium follows a power law distribution

$$\rho_a(r)=\eta_s r^{-s}. \quad (7)$$

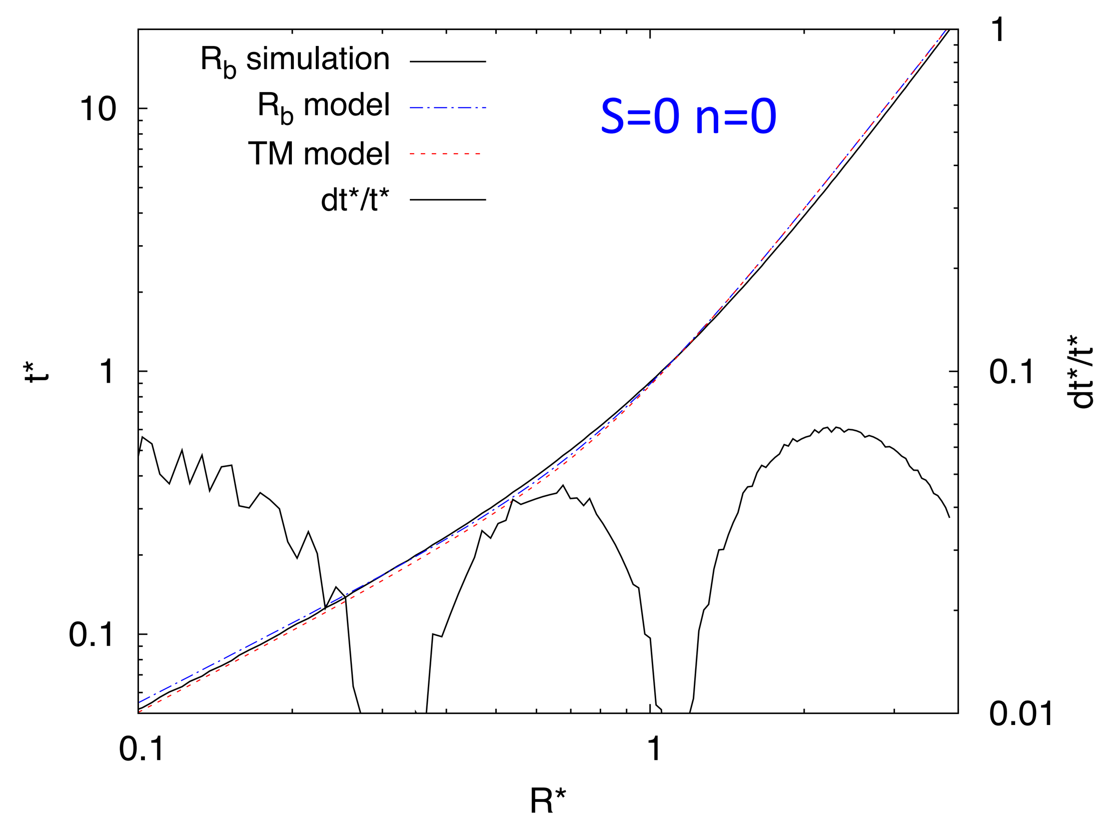
Now according to dimensional analysis, we could define the characteristic radius $R_{ch}=(M_{ej}/\eta_s)^{1/(3-s)}$ and time $t_{ch}=M^{(5-s)/2(3-s)}\eta_s^{1/(s-3)}E_{SN}^{-0.5}$ for the system. Denote quantity in the corresponding characteristic scale as $X^*=X/X_{ch}$, the analytical approximation for n<5 is found to be

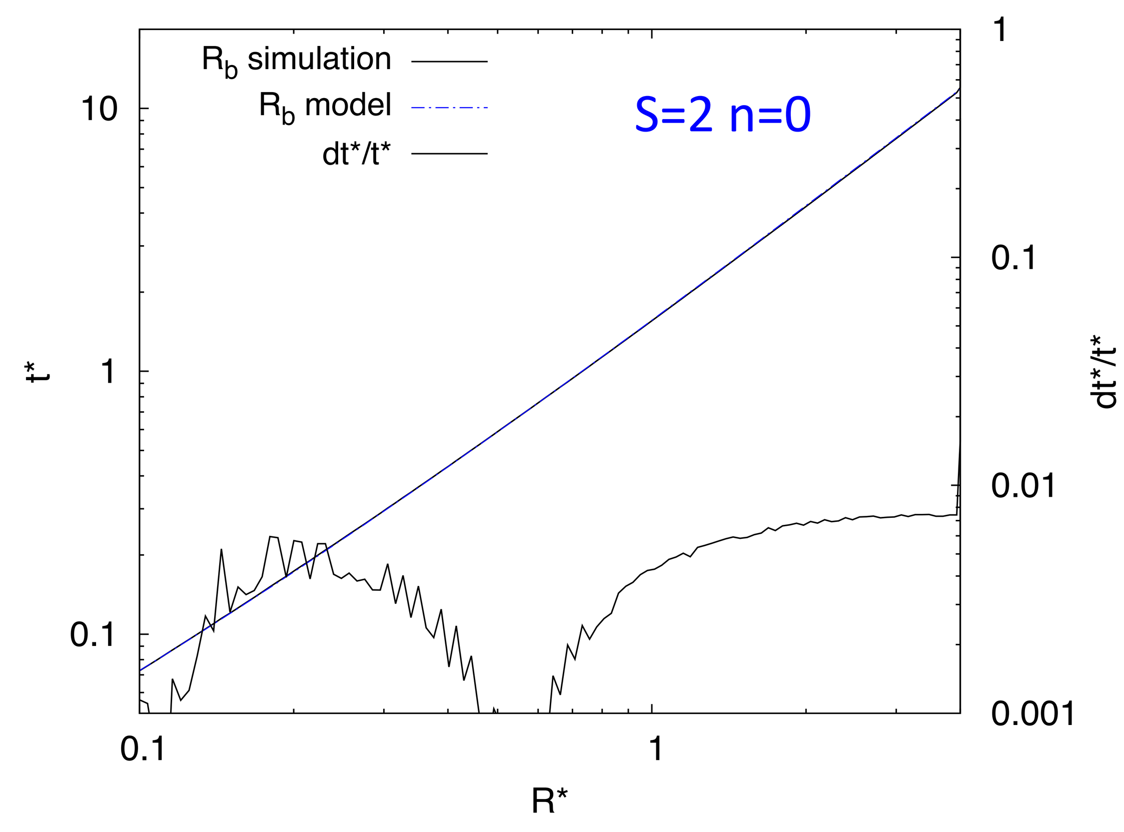
$$n < 5$$

$$t^*(R_b^*) = \left[\left(\frac{R_b^*}{\lambda}\right)^{2\alpha} + \left(\frac{R_b^{*5-s}}{\xi}\right)^{\alpha}\right]^{1/2\alpha}$$

where λ and ξ depend on the density distribution in ejecta and ambient medium respectively and could be obtained analytically. α is the only fitting parameter in the model.

Comparison with numerical simulation and Truelove&Mckee model

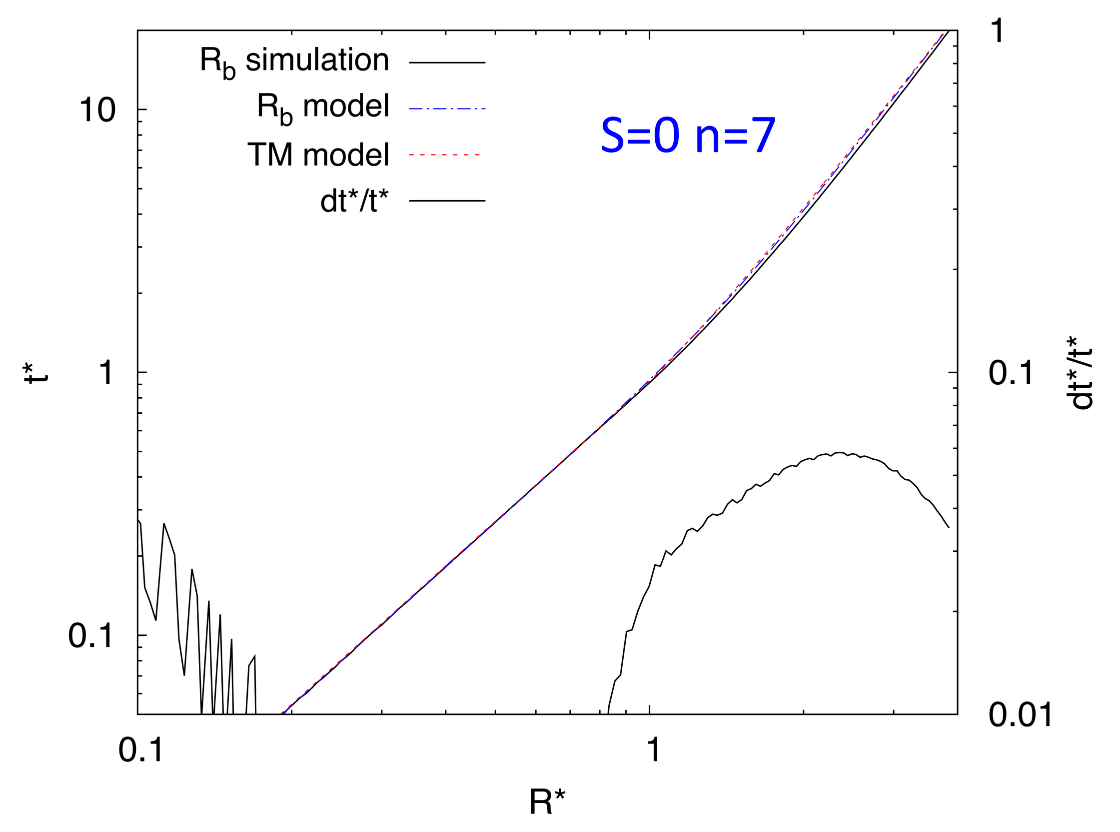


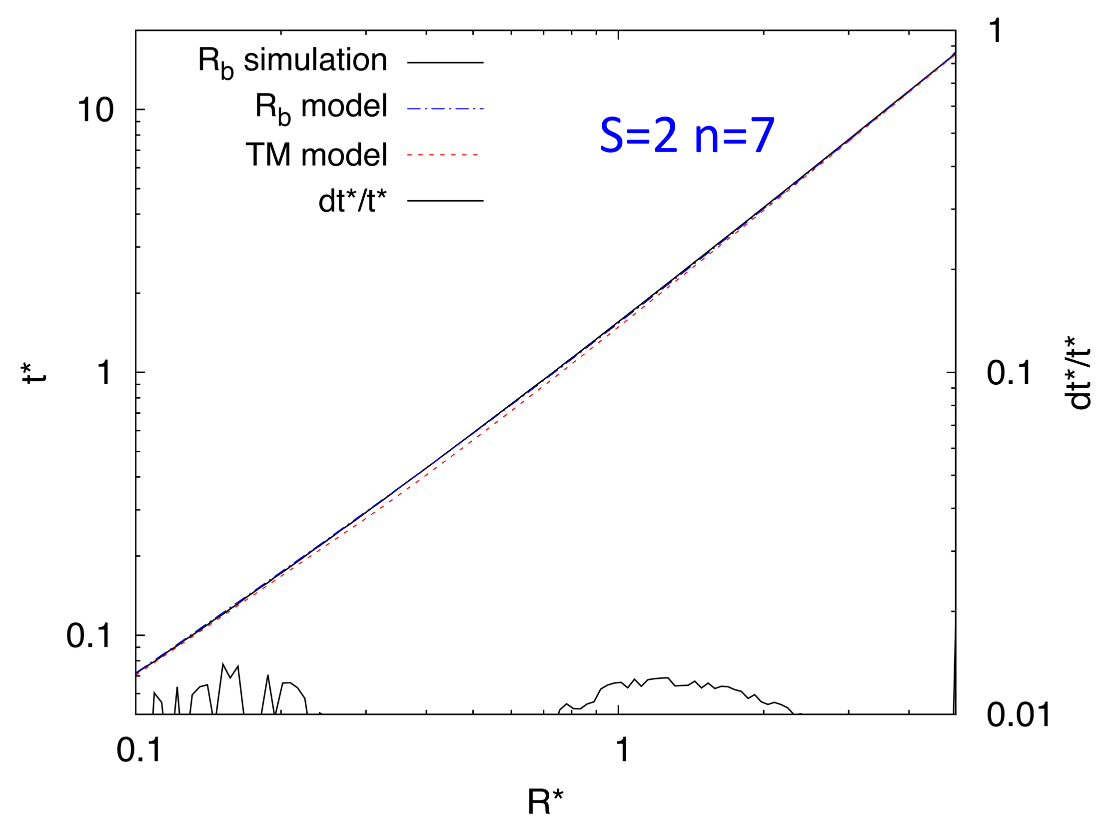


When n>5, a self similar driven wave (SSDW) solution exists during the transition from FE solution to ST solution (Chevalier 1982). The SSDW solution is equivalent to $\Pi_1^{(n-5)/2(n-s)}\,\Pi_2^{1/(n-s)}=\zeta$ where ζ is a dimensionless constant. For the time range interested in SNR evolution, only the transition from SSDW solution to ST solution is important while the evolution from FE solution to SSDW is so fast and not relevant. The analytical approximation now becomes

$$n > 5$$

$$t^*(R_b^*) = \left[\left(\frac{R_b^*}{\zeta}\right)^{2\alpha(n-s)/(n-3)} + \left(\frac{R_b^{*5-s}}{\xi}\right)^{\alpha}\right]^{1/2\alpha}$$





Summary and future work

In this poster, we present simple analytical approximation for the evolution of forward shock in a non-radiative SNR. The fitting results and uncertainty introduced by the model are summarized in the following 2 tables for both uniform ambient medium and wind density profile. Modeling for the reverse shock and possible extension to radiative phase are still under work.

Table 1: Basic parameters for analytical model and numerical simulation with $s=0$

n	α	ζ	t_0^*	t_{tran}^*	R_{tran}^*	$\Delta t^*/t^*$
0	1	-	10^{-3}	0.65	0.85	$\lesssim 7\%$
1	1	-	10^{-3}	0.56	0.80	$\lesssim 7\%$
2	0.8	-	10^{-3}	0.44	0.70	$\lesssim 7\%$
4	0.6	-	10^{-3}	0.08	0.33	$\lesssim 4\%$
6	2.5	1.06	3×10^{-4}	2.58	1.62	$\lesssim 12\%$
7	3.0	1.06	3×10^{-4}	1.82	1.41	$\lesssim 6\%$
8	3.0	1.08	3×10^{-4}	1.45	1.29	$\lesssim 6\%$
9	3.0	1.12	3×10^{-4}	1.25	1.21	$\lesssim 5\%$
10	3.0	1.15	3×10^{-4}	1.12	1.16	$\lesssim 5\%$
12	2.3	1.21	3×10^{-4}	1.00	1.09	$\lesssim 6\%$
14	2.0	1.26	3×10^{-4}	0.93	1.05	$\lesssim 6\%$

Table 2: Basic parameters for analytical model and numerical simulation with $s=2$

n	α	ζ	t_0^*	t_{tran}^*	R_{tran}^*	$\Delta t^*/t^*$
0	0.95	-	10^{-3}	0.11	0.14	$\lesssim 1\%$
1	0.9	-	10^{-3}	0.09	0.12	$\lesssim 1\%$
2	0.85	-	10^{-3}	0.05	0.08	$\lesssim 2\%$
4	0.65	-	10^{-3}	0.006	0.002	$\lesssim 3\%$
6	3.0	0.77	3×10^{-4}	1.3	0.86	$\lesssim 3\%$
7	3.0	0.83	3×10^{-4}	0.74	0.59	$\lesssim 2\%$
8	2.5	0.90	3×10^{-4}	0.5	0.45	$\lesssim 2\%$
9	2.0	0.97	3×10^{-4}	0.39	0.37	$\lesssim 2\%$
10	1.8	1.03	3×10^{-4}	0.32	0.32	$\lesssim 2\%$
12	1.5	1.14	3×10^{-4}	0.25	0.26	$\lesssim 1\%$
14	1.4	1.23	3×10^{-4}	0.21	0.23	$\lesssim 2\%$

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