# Synchrotron emission in the case of a partly random magnetic field, and the study of some general properties of radio shell-type SNRs.


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## SYNCHROTRON EMISSION (Classical Theory)

*For a homogeneous magnetic field* *(e.g. Rybicki & Lightmann 1979)*

- Single-particle ($\gamma$ fixed) synchrotron emission $\Rightarrow$ spectral powers $P_\perp$ and $P_\parallel$, respectively perpendicular and parallel to the projected magnetic field.
- The Stokes parameters can be then derived as:

$$\mathcal{I}'(\omega)=\frac{P_\perp+P_\parallel}{4\pi}=\frac{\sqrt{3}e^3B_\perp}{8\pi^2m_ec^2}F(x)\qquad \mathcal{Q}'(\omega)=\frac{P_\perp-P_\parallel}{4\pi}=\frac{\sqrt{3}e^3B_\perp}{8\pi^2m_ec^2}G(x)$$

where:

$$F(x)=x\int_x^\infty K_{5/3}(z)\,dz\qquad G(x)=xK_{2/3}(x)$$

modified Bessel functions of the 2nd type

and: $x=\dfrac{\omega}{\omega_c}=\dfrac{2}{3}\dfrac{m_ec}{eB_\perp}\dfrac{\omega}{\gamma^2}$

- For a power-law particle distribution $n(\gamma)=A\gamma^{-s}$

$$\mathcal{I}_{\rm PL}(\omega)=\frac{s+7/3}{s+1}W_oB_\perp^{(s+1)/2}\qquad \mathcal{Q}'_{\rm PL}(\omega)=W_oB_\perp^{(s+1)/2}$$

where: $W_o\propto\omega^{-(s-1)/2}$

so that: $\Pi_{\max}=\dfrac{\sqrt{\mathcal{Q}_{\rm PL}^2+\mathcal{U}_{\rm PL}^2}}{\mathcal{I}_{\rm PL}}=\left|\dfrac{\mathcal{Q}'_{\rm PL}}{\mathcal{I}'_{\rm PL}}\right|=\dfrac{s+1}{s+7/3}$ ($\simeq$70% for s $\simeq$2)

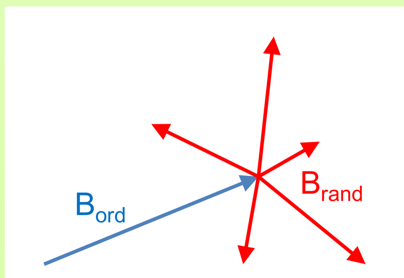
*Characteristics:* anisotropic, and strongly linearly polarized emission.

***While for a randomly oriented field***

- No polarization
- Isotropic emission (simply the orientation-averaged emissivity)

***BUT how to treat the combination of an ordered and a random field ?***

- Just sum up the two emissivities ?
  e.g. X % I($B_{\rm ordered}$) + Y % I($B_{\rm random}$) **NO**
- Or integrate the emissivities for the composed field over the whole probability distribution of the random component ? **OK**




## EXTENSION OF THE STANDARD THEORY TO TREAT A MIX OF ORDERED + RANDOM FIELD

For a field $\mathbf{B}=\bar{\mathbf{B}}+\delta\mathbf{B}$ (average value + isotropic Gaussian random fluctuations) an analytical solution is found:

$$\mathcal{P}_x(B_x)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{B_x^2}{2\sigma^2}\right)\qquad \mathcal{P}_y(B_y)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(B_y-\bar{B})^2}{2\sigma^2}\right)$$

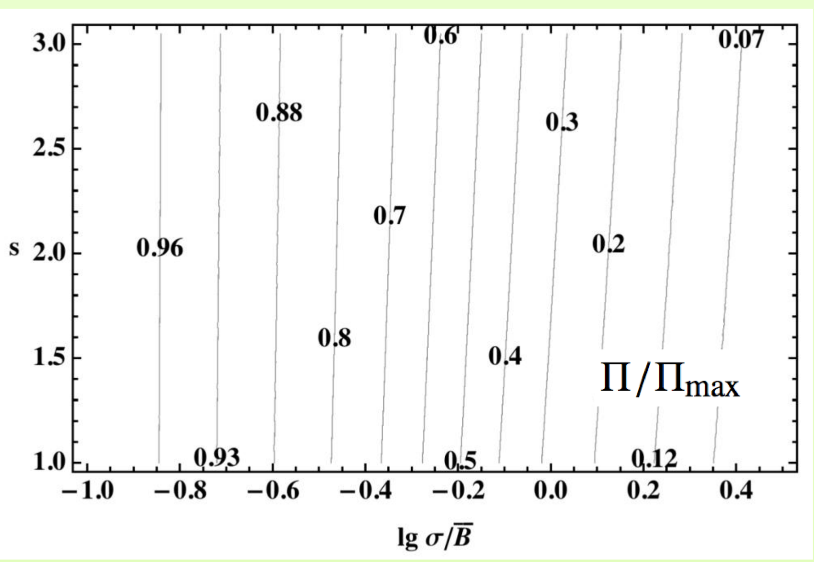
$$\langle\mathcal{I}\rangle_{\rm PL}=\mathcal{I}_{\rm PL}\left\{\Gamma\left(\frac{5+s}{4}\right)\left(\frac{\bar{B}}{\sqrt{2}\sigma}\right)^{-(1+s)/2}{}_1F_1\left(-\frac{1+s}{4},1,-\frac{\bar{B}^2}{2\sigma^2}\right)\right\}$$

$$\langle\mathcal{Q}\rangle_{\rm PL}=\mathcal{Q}_{\rm PL}\left\{\frac{1}{2}\Gamma\left(\frac{9+s}{4}\right)\left(\frac{\bar{B}}{\sqrt{2}\sigma}\right)^{(3-s)/2}{}_1F_1\left(\frac{3-s}{4},3,-\frac{\bar{B}^2}{2\sigma^2}\right)\right\}$$

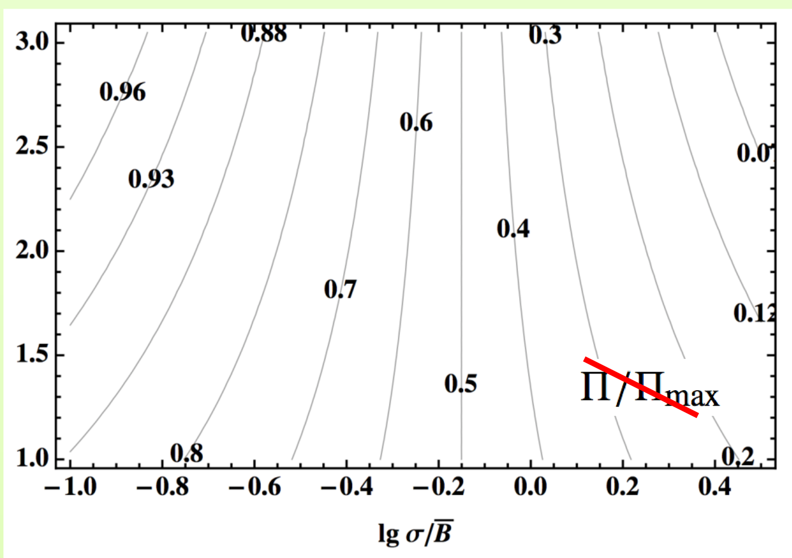
The factors in braces approach unity in the limit $\sigma\ll\bar{B}$, i.e. for a homogeneous magnetic field.

${}_1F_1(a,b,z)$ is the Kummer confluent hypergeometric function

*Map of the synchrotron polarization fraction (with the above approach)*



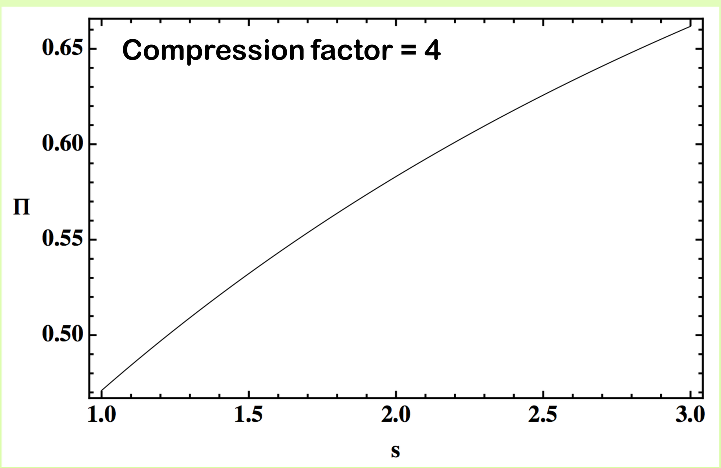

*Incorrect result obtained by simply adding an ordered and a fully random case*




- *Anisotropic distribution of the random field ( $\sigma_x\neq\sigma_y$ ), but no ordered field:* case valid for an efficient field amplification upstream of the shock, and subsequent shock compression. *(analytical solution)*

**Still large polarization fractions** (55-60 %)

- Why lower polarizations in radio SNRs?
- Just an effect of geometric projection ?




## FOR THE FUTURE

- Extending this technique to more general particle energy distributions (in order to treat the synchrotron cutoff seen in the X-rays).
- Including spatial correlations of the fluctuations (turbulence spectrum).
- Modelling the SNR with more detailed than thin-shell approximation.

**(SEE ALSO POSTER S.1.16)**


***REFERENCES:***

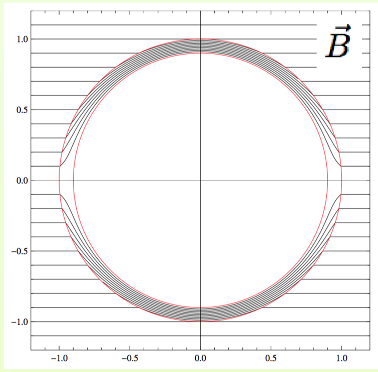
Bandiera & Petruk 2016, MNRAS 459, 178

Rothenflug et al. 2004, A&A 425, 121

Rybicki & Lightman 1979, "Radiative Processes in Astrophysics"


## APPLICATIONS TO A SIMPLIFIED MODEL OF SHELL-TYPE SUPERNOVA REMNANT

- Thin-shell approximation: although simplified, it allows a detailed modelling.
- Other assumptions:
  - fix the latitudinal dependence of the (meridional) field (e.g. expansion in a homogeneous ambient field) $\Rightarrow$
  - fix the latitudinal dependence of the particle injection




- Compute: - Emissivity, in the various Stokes parameters
  - Projection effects, depending on the aspect angle
  - Internal Faraday rotation (rotation of the polarization plane due to propagation effects)

**Intriguing effects of the internal Faraday rotation**

- While in the case of Faraday rotation from a foreground medium we simply have:

$$Q=\cos(2\beta)Q_{\rm i}-\sin(2\beta)U_{\rm i};\qquad U=\sin(2\beta)Q_{\rm i}+\cos(2\beta)U_{\rm i},\qquad \beta(z_1,z_2)=\frac{e^3\lambda^2}{2\pi m_e^2c^4}\int_{z_1}^{z_2}n(z')B_z(z')\,dz'.$$

so that a Rotation Measure parameter can be defined, by $\beta = RM\,\lambda^2$

- In the case of internal Faraday rotation, we now have instead:

$$Q=f_{\rm obs}\left(\cos(2RM_{\rm obs}\lambda^2)Q_{\rm i}-\sin(2RM_{\rm obs}\lambda^2)U_{\rm i}\right);\qquad U=f_{\rm obs}\left(\sin(2RM_{\rm obs}\lambda^2)Q_{\rm i}+\cos(2RM_{\rm obs}\lambda^2)U_{\rm i}\right),$$

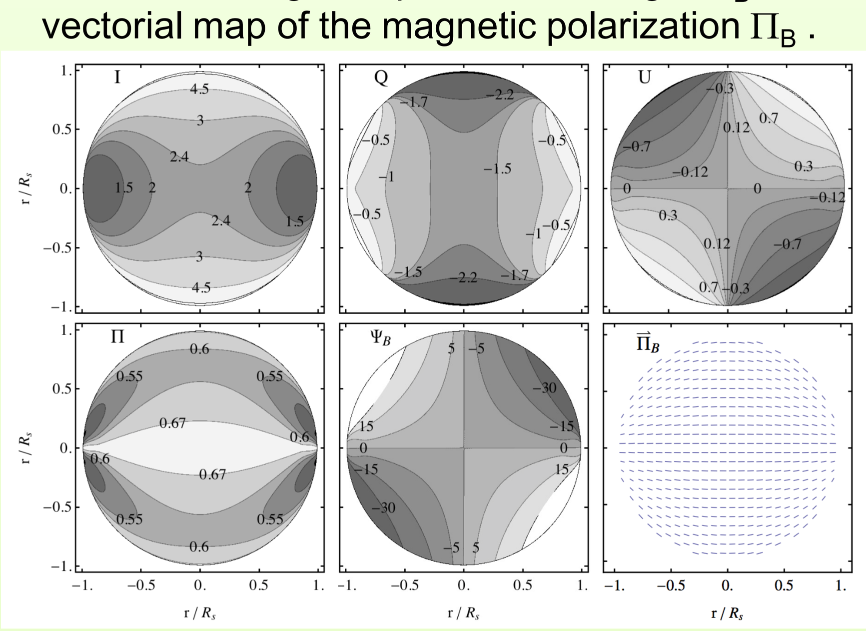
depolarization — now wavelength dependent

See Bandiera & Petruk (2016) for a generalized momentum treatment, in the short $\lambda$ limit.

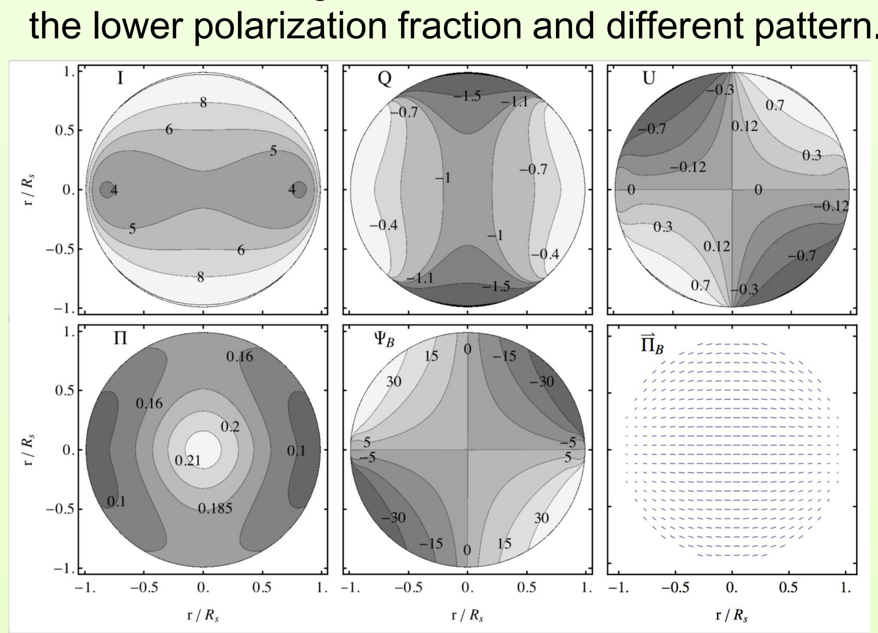
## SOME RESULTS

*Example: Isotropic particle injection; field as due to compression of the ambient field; aspect angle of 60°*

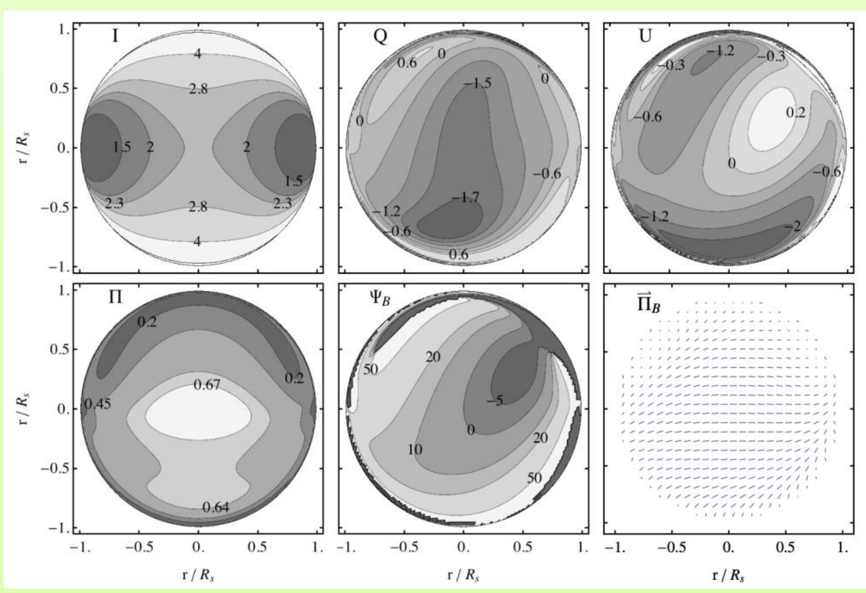
A. Maps of Stokes parameters I, Q, U, polarization fraction $\Pi$, magnetic polarization angle $\Psi_B$, and vectorial map of the magnetic polarization $\Pi_B$.



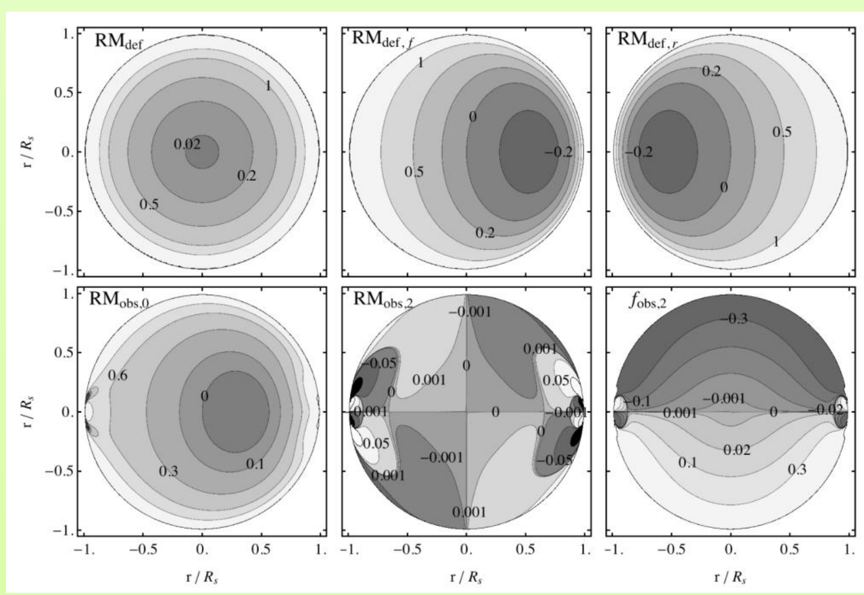

B. Same as Fig. A, but with the addition of a random field of same magnitude of the ordered field: note the lower polarization fraction and different pattern.




C. Same as Fig. A, but with a some Faraday rotation (of order unity, near the center) included.




D. Maps of quantities associated to Faraday rotation. First the rotation measure as from the standard definition, for the whole SNR, or just for the front ($f$) or rear ($r$) layer. Then the first power-law expansion terms of $RM_{obs}(\lambda^2)$ as well as the first non trivial terms of the depolarization factor $f_{obs}(\lambda^2)$.

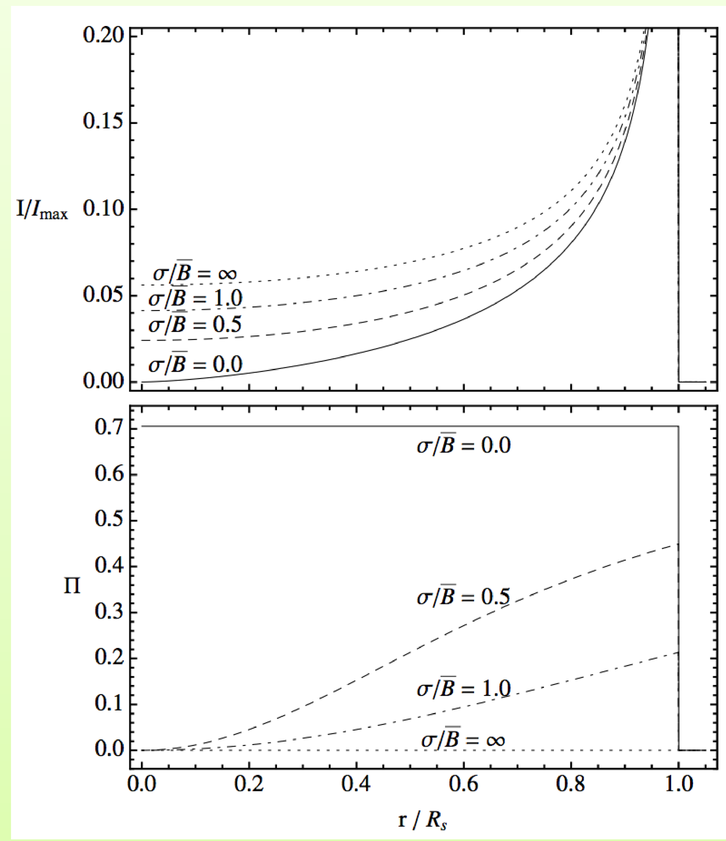


In general, detailed observations in radio of the internal Faraday rotation could provide information on the ordered MF structure.

## DIAGNOSTICS WITH RADIAL PROFILES IN RADIO

- Suggested as a tool to distinguish a polar-cap structure from a barrel-like one (Rothenflug et al. 2004).
- Low surface brightness near the projected center as incompatible with a barrel-like SNR structure.
- But isotropic emission is implicitely assumed.

Analysis of case where the ordered MF has a barrel like large-scale structure plus a small-scale radial MF pattern, compatible with polarization in young SNRs & justified by Rayleigh-Taylor like instabilities. The radial profiles of intensity and polarization are shown, for different levels of the random field. Maximum $\Pi$ levels ~40% need at most $\sigma/\bar{B}\simeq 0.5$, which implies sensibly lower central emission with respect to what estimated by assuming isotropic emission.




**Therefore the criterion introduced by Rothenflug et al. 2004 needs to be revised.**